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Trajectories and particle creation and annihilation in quantum field theory

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Abstract

We develop a theory based on Bohmian mechanics in which particle world lines can begin and end. Such a theory provides a realist description of creation and annihilation events and thus a further step towards a 'beable-based' formulation of quantum field theory, as opposed to the usual 'observable-based' formulation which is plagued by the conceptual difficulties—such as the measurement problem—of quantum mechanics.

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1. Introduction

A formulation of nonrelativistic quantum mechanics based on objectively existing particle positions and particle trajectories, now usually called Bohmian mechanics, was proposed 50 years ago by Bohm (and even earlier by de Broglie); see [4] for a recent overview. Today there remain two big challenges for this approach: to form a relativistic version and a version suitable for quantum field theory (QFT). Here we shall address the latter. We describe a general and particularly natural way of extending Bohmian mechanics to QFTs, explicitly giving the equations for a simple example.

Bohm himself proposed [2, p 230] that Bohmian mechanics should be extended to QFT by means of the incorporation of the actual field configuration, guided by a wavefunctional (the state vector). In contrast, Bell proposed a model [1, p 173] in which, instead of the field configuration, the local beables are the fermion numbers at each site of a lattice discretizing 3-space. We argue that it is instructive to modify Bell's proposal in two ways, and thus get a similar but even simpler theory with a direct connection to Bohmian mechanics. Below, we give an explicit example of such a theory for a particularly simple Hamiltonian.

Bell's model contains no beables representing the bosonic degrees of freedom (such as radiation), neither an actual field, nor actual particles, nor anything else; the existence of a radiation part of the state vector is relevant only to the behaviour of the fermions. This is certainly consistent and empirically irrefutable, but it is neither *necessary* (as our example below shows), nor even a *natural* view[5]. In the model proposed below, the bosons have the same status as the fermions: they are particles, described by their positions. This, of course, should not be regarded as discouraging consideration of the approach based on actual field configurations.

The other deviation from Bell's proposal is the replacement of the lattice by continuous space. The lattice was introduced in the first place for the purpose of providing an effective ultraviolet cutoff and thus a well-defined Hamiltonian. Such a cutoff, however, can also be realized by smearing out the interaction Hamiltonian through convolution with, say, a sharply peaked but bounded function φ. The continuum analogue of the particle number at every lattice site is the position of all particles in ordinary space, with the total number of particles $N(t)$ possibly varying with time. However, in the model we propose the particles follow Bohmian trajectories, except when there is particle creation or annihilation. (For a discussion of how Bohmian mechanics arises from a lattice model (in the absence of interaction) in the limit of vanishing lattice width, see [8, 9].)

Our proposal profits, we believe, from making this contact with Bohmian mechanics, since then every argument for taking the Bohmian trajectories seriously also provides some support for the proposal. Bohmian mechanics also profits from this contact because the Bohmian trajectories *can* then be taken seriously even in the framework of QFT.

Our moving configuration $Q(t)$ is constructed in such a way that it is random with distribution at time t equal to $\rho(t) = |\Psi(t)|^2$, where $\Psi(t)$ is the (position) Fock space representation of the state vector $|\Psi(t)\rangle$ at time t. (For a model in which there are only bosons, the n-particle component of Ψ is

$$\Psi^{(n)}(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n) = \frac{1}{\sqrt{n!}} \langle 0 | a(\boldsymbol{x}_1) \cdots a(\boldsymbol{x}_n) | \Psi \rangle \tag{1}$$

where $|0\rangle$ is the Fock vacuum and $a(\boldsymbol{x}) = (2\pi)^{-3/2} \int \mathrm{d}^3\boldsymbol{k}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} a_{\boldsymbol{k}}$ is the boson annihilation operator at position $\boldsymbol{x}$.) In particular, the probability that $N(t) = n$, i.e., there are n particles at time t, equals the integral of $\rho(t)$ over $K^{(n)} \cong (\mathbb{R}^3)^n$ (suitably symmetrized), the n-particle sector of configuration space; i.e., the L^2 norm of the projection of $|\Psi\rangle$ onto the n-particle subspace of Fock space. A nontrivial superposition of quantum states with different particle numbers leads to a probability distribution over different particle numbers, among which only one is, of course, actually realized.

2. Configuration jumps

The Hamiltonian for QFTs is typically a sum of terms, each of which yields a contribution to the motion we wish to propose. It is quite generally the case that $H = H_0 + H_{\mathrm{int}}$ where the free Hamiltonian H_0 corresponds naturally to a deterministic motion—in the model considered here that of Bohmian mechanics, and in a relativistic model for example the Bohm–Dirac motion [2, p 274]—given by a (time-dependent) velocity field $v = v^{\Psi(t)}(q, t)$, on configuration space

[5] After all, if the role of the wavefunction is to guide particles, it would seem that there should be as many particles as there are variables in the wavefunction. In addition, the Hilbert space of the radiation degrees of freedom (unlike that for quark colour) is not a purely abstract Hilbert space, but is related to spacetime points (via creation and annihilation operators, for example), a fact that would seem surprising if the state vector were not related to spacetime objects such as particles, strings or fields.

$K = \cup_n K^{(n)}$, that defines the 'deterministic part' of the process, while the interaction term H_{int} corresponds to random jumps, a continuum version of the process proposed by Bell [1, p 173], defined by jump rates $\sigma = \sigma(q', q, t) = \sigma^{\Psi(t)}(q', q)$ for a transition from q to q' at time t. (This means that when the actual configuration Q at time t is q, then with probability (density, with respect to q') $\sigma(q', q, t)\,\mathrm{d}t$, Q will jump from (very near) q to q' in the time-interval $(t, t + \mathrm{d}t)$.)[6]

The relevant continuum analogue of Bell's jump rates for $H = H_{\text{int}}$, equations (6)–(8) of [1, p 173], is

$$\sigma^{\Psi}(q', q) = \frac{2}{\hbar} \frac{(-\mathrm{Im}\overline{\Psi(q)}\langle q|H_{\text{int}}|q'\rangle\Psi(q'))^{+}}{\overline{\Psi(q)}\Psi(q)} \tag{2}$$

where for particles with spin the two products in the numerator and the product in the denominator are local spinor inner products, and where we have used the notation $A^{+} = \max(A, 0)$ for the positive part of $A \in \mathbb{R}$. This will typically be well defined (as jump rates), since the kernel $\langle q|H_{\text{int}}|q'\rangle$ of the (cutoff) interaction Hamiltonian of a QFT should involve nothing worse than δ-function singularities.

The complete process, corresponding to the total Hamiltonian, is then given by the deterministic motion with velocity v, randomly interrupted by jumps, with rate σ, after each of which the deterministic motion is resumed until it is again interrupted. As a function of t, each realization $Q(t)$ is thus piecewise smooth. At the end of a smooth piece, Q jumps to the starting point of the next smooth piece. What is stochastic about Q are the times at which the jumps take place and the destinations of the jumps. The probabilities for times and destinations are governed by the wavefunction. $Q(t)$ is a Markov process.

As described, our process is so designed as to directly imply the following equivariance theorem: *if $Q(t_0)$ is chosen at random with distribution $|\Psi(t_0)|^2$, then at every later time $t > t_0$, $Q(t)$ is distributed with density $|\Psi(t)|^2$.* This can also be explicitly checked by comparing the equation for $\partial(\overline{\Psi}\Psi)/\partial t$ as implied by the evolution equation of the quantum state, $\mathrm{i}\hbar\,\partial\Psi/\partial t = H\Psi$, and the master equation for the distribution of $Q(t)$, which reads

$$\frac{\partial}{\partial t}\rho(q, t) = -\nabla \cdot (\rho(q, t)v(q, t)) + \int_K \mathrm{d}q'(\rho(q', t)\sigma(q, q', t) - \rho(q, t)\sigma(q', q, t)).$$

The choice of jump rates that will make the $|\Psi|^2$ distribution equivariant is not unique. But the 'minimal' jump rates are unique, and these are the ones we have chosen. Details on the choice of jump rates as well as an elucidation of the general mathematical structure underlying equivariant processes will be presented in [3].

Corresponding to the standard interaction terms in QFT, the possible jumps are very restricted: there are only changes in the particle number by ± 1, and possible types are (i) appearance or (ii) disappearance of a particle (while the others remain at their positions) or (iii) replacement of a particle by two others or (iv) the reverse of this. Type (i) is appropriate for, e.g., photon emission, (ii) for absorption, (iii) for pair creation and (iv) for pair annihilation. (It follows that there is no discontinuity in the individual particle world lines, in spite of the discontinuity in Q.) In our explicit model, only types (i) and (ii) occur.

3. An explicit model

We now present an explicit theory. It is based on a 'baby' QFT taken from [6, p 339] and [5], containing two species of particles, which we simply call electrons and photons. Electrons,

[6] One can regard the jumps as having two ingredients: a jump occurs with total rate $\bar{\sigma}(q, t) = \int \sigma(q', q, t)\,\mathrm{d}q'$; when a jump does occur, the destination is randomly chosen with distribution $\sigma(q', q, t)/\bar{\sigma}(q, t)$.

whose number stays constant, emit and absorb photons. To keep things simple, we employ nonrelativistic dispersion relations (as everything here is nonrelativistic) for both electrons and photons, and give the photon a positive (rest) mass. In addition, we ignore spin and polarization, and, of course, smoothen the interaction Hamiltonian.

In field theoretic language, we have a bosonic field $\phi(\boldsymbol{x}) = a^\dagger(\boldsymbol{x}) + a(\boldsymbol{x})$, with $a^\dagger$ and a the photon creation and annihilation operators (cf equations (4) and (5)) and a fermionic field $\psi(\boldsymbol{x})$, and the Hamiltonian is the sum

$$\begin{aligned} H &= H_{\mathrm{F}} + H_{\mathrm{B}} + H_{\mathrm{int}} \\ &= (1/2m_{\mathrm{F}}) \int \mathrm{d}^3\boldsymbol{x} \nabla\psi^\dagger(\boldsymbol{x})\nabla\psi(\boldsymbol{x}) + (1/2m_{\mathrm{B}}) \int \mathrm{d}^3\boldsymbol{x} \nabla a^\dagger(\boldsymbol{x})\nabla a(\boldsymbol{x}) \\ &\quad + g \int \mathrm{d}^3\boldsymbol{x} \psi^\dagger(\boldsymbol{x})\phi_\varphi(\boldsymbol{x})\psi(\boldsymbol{x}) \end{aligned}$$

where $\phi_\varphi(\boldsymbol{x}) = \int \mathrm{d}^3\boldsymbol{y}(\varphi(\boldsymbol{x}-\boldsymbol{y})a^\dagger(\boldsymbol{y}) + \overline{\varphi}(\boldsymbol{x}-\boldsymbol{y})a(\boldsymbol{y}))$ is the cutoff bosonic field, m_{F} and m_{B} denote the mass of the electrons and photons, and g is a real coupling constant. H commutes with the fermion number operator $N_{\mathrm{F}} = \int \mathrm{d}^3\boldsymbol{x}\psi^\dagger(\boldsymbol{x})\psi(\boldsymbol{x})$.

Since the fermion number is conserved, we give it a fixed value N. The configuration space (changing notation slightly from before) is $K = \bigcup_{m=0}^{\infty} K^{(m)}$ where m is the photon number and

$$K^{(m)} := (\mathbb{R}^3)^N \times (\mathbb{R}^3)^m \qquad K^{(0)} = (\mathbb{R}^3)^N.$$

We will denote the electron coordinates by $x := x^{(N)} := (\boldsymbol{x}^1, \ldots, \boldsymbol{x}^N)$ and the photon coordinates by $y := y^{(m)} = (\boldsymbol{y}^1, \ldots, \boldsymbol{y}^m)$. The full configuration is thus given by $q = (x, y)$ and, more explicitly, by $q^{(m)} = (x, y^{(m)})$.

In terms of the wavefunction Ψ (the (position) Fock representation of the quantum state), the Hilbert space inner product is

$$\langle\Phi|\Psi\rangle := \sum_{m=0}^{\infty} \int_{K^{(m)}} \mathrm{d}^{3N}x\, \mathrm{d}^{3m}y\, \overline{\Phi}(x, y)\Psi(x, y)$$

and the contributions to the Hamiltonian assume the form

$$H_{\mathrm{F}} = -\sum_i \frac{\hbar^2}{2m_{\mathrm{F}}}\Delta_i \qquad H_{\mathrm{B}} = -\sum_j \frac{\hbar^2}{2m_{\mathrm{B}}}\Delta_j$$

and

$$H_{\mathrm{int}} = g\sum_{i=1}^{N} \phi_\varphi(\boldsymbol{x}^i) = g\sum_{i=1}^{N} \left(a_\varphi^\dagger(\boldsymbol{x}^i) + a_{\overline{\varphi}}(\boldsymbol{x}^i)\right) \tag{3}$$

with creation and annihilation operators $a_\varphi^\dagger$ and $a_{\overline{\varphi}}$ acting on Fock space in a smeared-out form,

$$a_\varphi^\dagger(\boldsymbol{x}) = \int \mathrm{d}^3\boldsymbol{u}\varphi(\boldsymbol{u}-\boldsymbol{x})a^\dagger(\boldsymbol{u}) \qquad a_{\overline{\varphi}}(\boldsymbol{x}) = \int \mathrm{d}^3\boldsymbol{u}\overline{\varphi}(\boldsymbol{u}-\boldsymbol{x})a(\boldsymbol{u})$$

where

$$(a^\dagger(\boldsymbol{u})\Psi)(q^{(m)}) = \frac{1}{\sqrt{m}}\sum_j \delta(\boldsymbol{y}^j - \boldsymbol{u})\Psi(\widehat{q^j}) \tag{4}$$

$$(a(\boldsymbol{u})\Psi)(q^{(m)}) = \sqrt{m+1}\,\Psi(q^{(m)}, \boldsymbol{u}). \tag{5}$$

$a_\varphi^\dagger(\boldsymbol{x})$ creates a new photon in state φ centred at $\boldsymbol{x} \in \mathbb{R}^3$ (which will be the position of an electron), and $a_{\overline{\varphi}}(\boldsymbol{x})$ annihilates a photon with 'form factor' φ at $\boldsymbol{x}$.[7] Here $(q^{(m)}, \boldsymbol{u})$ is the configuration with a photon at $\boldsymbol{u}$ added to $q^{(m)}$ and $\widehat{q^j}$ is the configuration with the jth photon deleted from $q^{(m)}$. Thus

$$(H_{\text{int}}\Psi)(q^{(m)}) = \frac{g}{\sqrt{m}} \sum_{i=1}^{N} \sum_{j=1}^{m} \varphi(\boldsymbol{y}^j - \boldsymbol{x}^i)\Psi(\widehat{q^j}) + g\sqrt{m+1} \sum_{i=1}^{N} \int \mathrm{d}^3\boldsymbol{y}' \overline{\varphi}(\boldsymbol{y}' - \boldsymbol{x}^i)\Psi(q^{(m)}, \boldsymbol{y}'). \tag{6}$$

Ψ satisfies the Pauli principle, i.e., it is symmetric in the photon variables and antisymmetric in the electron variables.

We now turn to the particles, described by the actual electron configuration X and the actual photon configuration Y. The deterministic part of the motion $Q(t) = (X(t), Y(t))$, corresponding to $H_{\mathrm{F}} + H_{\mathrm{B}}$, is the usual Bohm motion,

$$\dot{X}^i = v^\Psi_{\mathrm{F},i}(Q) := \frac{\hbar}{m_{\mathrm{F}}} \operatorname{Im} \frac{(\partial\Psi/\partial\boldsymbol{x}^i)(Q)}{\Psi(Q)} \tag{7}$$

$$\dot{Y}^j = v^\Psi_{\mathrm{B},j}(Q) := \frac{\hbar}{m_{\mathrm{B}}} \operatorname{Im} \frac{(\partial\Psi/\partial\boldsymbol{y}^j)(Q)}{\Psi(Q)} \tag{8}$$

and it follows from (2) and (6) that only two kinds of jumps occur, with rates as follows:

- The jth photon vanishes, while all the other particles stay at their positions. Thus $q' = \widehat{q^j}$ and the jump rate from $q = q^{(m)}$ to q' is
$$\sigma(q', q) = \frac{2g}{\hbar\sqrt{m}} \left[-\sum_{i=1}^{N} \operatorname{Im} \frac{\Psi(\widehat{q^j})\varphi(\boldsymbol{y}^j - \boldsymbol{x}^i)}{\Psi(q)} \right]^+. \tag{9}$$
- A new photon appears at $\boldsymbol{y}'$, while all the other particles stay at their positions. Thus $q' = (q^{(m)}, \boldsymbol{y}')$ and the jump rate from $q = q^{(m)}$ to q' (a density in the variable $\boldsymbol{y}'$) is
$$\sigma(q', q) = \frac{2g}{\hbar}\sqrt{m+1} \left[-\sum_{i=1}^{N} \operatorname{Im} \frac{\Psi(q, \boldsymbol{y}')\overline{\varphi}(\boldsymbol{y}' - \boldsymbol{x}^i)}{\Psi(q)} \right]^+. \tag{10}$$

We reiterate that these choices guarantee that the process obeys the equivariance theorem mentioned in the previous section.

Note that, contrary to what might have been expected, the creation operator in the interaction Hamiltonian (3) corresponds, according to (2), to the annihilation rate, while the annihilation operator corresponds to the creation rate. There is less to this than meets the eye: the correspondence is an artefact of the way (2) is written, and if (2) had been written equivalently as

$$\sigma^\Psi(q', q) = \frac{2}{\hbar} \frac{(\operatorname{Im} \overline{\Psi(q')}\langle q'|H_{\text{int}}|q\rangle\Psi(q))^+}{\overline{\Psi(q)}\Psi(q)} \tag{11}$$

the correspondence would have been reversed. Note also that if φ is supported by a δ-ball around the origin of $\mathbb{R}^3$, then photons can be created or annihilated only in the δ-neighbourhood of an electron.[8]

[7] If the form factor φ is square-integrable (as we assume) it provides an ultraviolet cutoff; it can be regarded as determining the effective range of an electron's power to create or absorb a photon.

[8] One might suspect that if δ were small, there would be only a small probability (perhaps of order δ^3) that a photon will come closer than δ to an electron. But equivariance implies that this is not so: if a certain amount of $|\Psi^{(m)}|^2$ flows to the sector $K^{(m-1)}$, then the probability for a photon to be annihilated is just as large.

Our model is translation invariant (unless one introduces an additional external potential), time translation invariant, rotation invariant provided φ is spherically symmetric, time reversal invariant provided φ is real valued, and gauge invariant provided one introduces an external vector potential—in all derivatives acting on electron coordinates—and an external scalar potential into H_{F}. Galilean boost invariance fails, but not because of the Bohmian variables; owing to the interaction term (6), it fails, in fact, for the evolution of the quantum state. The reason is that, roughly speaking, a photon gets created with wavefunction φ which cannot be Galilean invariant. Alternatively, one could say that under boosts with velocity $\boldsymbol{u}$ the form factor $\varphi(\boldsymbol{y}-\boldsymbol{x})$ must be replaced by $\exp\left(\mathrm{i}m_{\mathrm{B}}\boldsymbol{u}\cdot\boldsymbol{y}/\hbar\right)\varphi(\boldsymbol{y}-\boldsymbol{x})$. Note that even with the cutoff removed, i.e. for $\varphi(\boldsymbol{y}) = \delta(\boldsymbol{y})$, the quantum dynamics is not Galilean invariant.

If we add to H a suitable confining potential (and φ is real valued, and the rest energy of the photon is made positive), it possesses a unique ground state [7], and this ground state (as every nondegenerate eigenstate) is real up to an overall phase. Thus in this state the jump rates (9) and (10) as well as the velocities (7) and (8) vanish identically—nothing moves. Surprisingly, perhaps, the Bohmian particles do not perform any 'vacuum fluctuations'.

As in Bohmian mechanics, disentangled subsystems are governed by the same laws as for the whole. More precisely, if Φ and Ψ are two wavefunctions from Fock space having disjoint supports S_Φ and S_Ψ in physical space $\mathbb{R}^3$, $\Phi\otimes\Psi$ defines another Fock state $\Phi\odot\Psi$ after suitable symmetrization; if the distance between S_Φ and S_Ψ is at least the diameter δ of the support of φ, then in a system with wavefunction $\Phi\odot\Psi$ the particles in S_Φ and those in S_Ψ will not influence each other, each set moving independently according to the corresponding 'factors'. This is a consequence of the fact that the velocities and jump rates are homogeneous of degree 0 in the wavefunction.

While the other particles keep their positions when a photon is created or annihilated, their velocities may change discontinuously because (7) evaluated at the destination may differ from (7) evaluated at the point of departure. As a result, the world lines of all (possibly distant) particles, if the particles are entangled, will have kinks at the times of particle creation or annihilation. These kinks will however not be visible in, say, a cloud chamber since the necessary entanglement is destroyed by the decoherence of the tracked particle with its environment, caused, say, by the particle's interaction with the vapour.

4. Removing the cutoff

Removing the cutoff is of course problematical, which is why the cutoff was introduced in the first place. However, the problems arise primarily from the evolution equation of the wavefunction, not those of the Bohmian configuration, equations (7) through (10). Suppose that a family of φ is parametrized by $\Lambda\in\mathbb{R}$, and that as $\Lambda\to\infty$, $\varphi^\Lambda(y)$ approaches $\delta(y)$. Then the limit $\Lambda\to\infty$ corresponds to 'removing the cutoff'. Unfortunately, the corresponding H^Λ will not converge in any reasonable sense. But there exist numbers E^Λ tending to infinity [5] such that $H^\Lambda - E^\Lambda$ does converge in a suitable sense, and the evolution of the wavefunction is well defined. This seems completely acceptable. We do not know whether the corresponding process Q^Λ approaches (given a fixed initial wavefunction) a limiting process Q^∞. Deciding this requires a careful mathematical study, but at least we see nothing precluding this possibility: the velocity law is not affected by the cutoff, while absorption might become deterministic in the limit $\Lambda\to\infty$ and occurs whenever (and only when) a photon hits an electron.

Other Hamiltonians, more sophisticated ones, are more problematical, and do not possess a limit as $\Lambda\to\infty$, even after subtracting an 'infinite energy'. In some cases, this is due to the creation of a large (average) number m^Λ of photons that goes to infinity with Λ. On the other hand, it is not clear that removing the cutoff is desirable or relevant. That is, there might exist an effective UV cutoff in nature, just as there is an effective IR cutoff (the finite radius of the universe).

Be that as it may, if the unitary evolution on some Hilbert space does not survive the limit $\Lambda \to \infty$, we face a problem, one that seems particularly bad for Bohmian theories, which so heavily rely on the wavefunction and, consequently, its having a well-defined unitary evolution. But appearances are misleading here. From a Bohmian viewpoint, the basic variable, bearing all the physical implications of the theory, is the configuration Q, whereas Ψ and H are only theoretical objects whose purpose is to generate a law of motion for Q. And, indeed, a law for Q might arise as a limit $\Lambda \to \infty$ of the law induced by Ψ^Λ and H^Λ; it might be the case that while Ψ^Λ and H^Λ do not have a limit, the time evolution of Q is well defined in the limit. After all, this is precisely what occurs when one considers the limit $\hbar \to 0$ of nonrelativistic quantum mechanics: while the Hamiltonian and wavefunction do not have any sensible limit, the law for Q does!

5. Determinism

We close with a remark on (the lack of) determinism. It may seem surprising that we abandon determinism. Was not the main point of hidden variables to restore it? Actually, no. What was important was to provide a clear and coherent account of quantum mechanics. The simplest such account, we believe, is provided by Bohmian mechanics, which happens to be deterministic. And the simplest such account of QFT seems to be of the sort we have presented here, which is stochastic.

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